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Nonlinear Dynamic Response of Piezoelectric Plates Considering Damage Effects

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Absti act: Based on the Talreja's tensor valued internal state variables damage model and the Helmhotlz free energy of piezoelectric material, the constitutive relations of the piezoelectric plates with damage are derived. Then, the nonlinear dynamic equations of the piezoelectric plates considering damage are established. By using the finite difference method and the Newmark scheme, these equations are solved and the effects of damage and electric loads on the nonlinear dynamic response of piezoelectric plates are discussed.

Introduction

The research of piezoelectric laminated structures has received considerable attention in recent years. Some investigations on the vibration of piezoelectric laminated plates have been done. Based on classical plate theory, Wang and Rogers[1] presented a model for laminated plates with spatially distributed piezoelectric patches. Tzou and Gadre [2] analyzed thin laminates shell with actuators for distributed vibration control. Xu et.al.[3] studied the free vibration of piezothermoelectric laminated plate based on the 3D theory. Mitchell and Reddy [4] proposed the theory of the piezoelectric. laminated plates by using classical plate theory and simple third-order theory, respectively. A.Benjeddou et.al.[5] studied the free vibration of simply-supported piezoelectric adaptive plates by adopting an exact sandwich formulation. However, various damage will emerge easily in the piezoelastic laminated structures. And the emergence and evolution of the damage will reduce the stiffness of structures and lead to the change of mechanical behaviors. But, there is only little research on the dynamic behaviors of the piezoelectric plate including damage effects.

In the present study, the constitutive relations of the piezoelectric plates with damage are derived, the nonlinear dynamic equations of the piezoelectric plate considering damage are established and solved by adopting the finite difference method and the Newmark scheme. Numerical results show that the effects of damage and electric loads on the nonlinear dynamic response of piezoelectric plates.

Basic Equations

Consider a piezoelectric rectangular plate having thickness h, length a and width b. The plate is subjected to dynamic transverse load $q(x,y,t)$ combined with electrical load.

Talreja adopted a second-order tensor to describe the damage. Supposing the k th crack in the characteristic volume is characterized by a symmetric second-order tensor $\omega^{(k)}$ [6]. Helmholtz free energy of piezoelectric material can be written as the invariant function of elastic strains, the electric field intensity and damage variables. Considering a piezoelectric single-layer plate, the local coordinate system $O-123$ is selected, in which axis $1,2$ denote the two principal direction of the piezoelectric plate, axis 3 is vertical to the midsurface. According to the Kirchhoff hypothesis for plate ($\varepsilon_{13}=\varepsilon_{23}=0$) and applying Voigt notation to describe strains and damage variables, the bases of invariant can be written as $\varepsilon_1,\varepsilon_2,\varepsilon_3$, $\varepsilon_6^{\ 2}$, $\omega_1^{\ k},\omega_2^{\ k},\omega_3^{\ k}$, $(\omega_4^{\ k})^2,(\omega_5^{\ k})^2,(\omega_6^{\ k})^2$, $\omega_4^{\ k}\omega_5^{\ k}\omega_6^{\ k}$, $\varepsilon_6\omega_6^{\ k},\varepsilon_6\omega_4^{\ k}\omega_5^{\ k}(k=1,2,\cdots\cdots n)$, E_1,E_2,E_3, where n is the number of the cracks' direction in the material.

For the small strain problems, Helmholtz free energy can be expressed as a quadratic expression of the strains or the electric field intensity, a mixture quadratic expression of strains and electric field intensity and a linear expression of damage variables. Then the stresses and the electric displacements can be expressed as

$$\sigma_p = \frac{\partial(\rho\psi)}{\partial\varepsilon_p} = [C_{pq}^0 + \sum_{k=1}^{n} C_{pq}^k]\varepsilon_q - [e_{pm}^0 + \sum_{k=1}^{n} e_{pm}^k]E_m,\quad D_m = -\frac{\partial(\rho\psi)}{\partial E_m} = [e_{qm}^0 + \sum_{k=1}^{n} e_{qm}^k]^T\varepsilon_q + [\kappa_m^0 + \sum_{k=1}^{n}\kappa_m^k]E_m \,. \quad (1)$$

where $\rho\psi$ is the Helmholtz free energy of piezoelectric material, C_{pq}^0 is the material constants without damage, κ_m^0 is the piezoelectric constants without damage, e_{qm}^0 is the permittivity matrix constants without damage, C_{pq}^k is the material constants with damage, κ_m^k is the piezoelectric constants with damage, e_{qm}^k is the permittivity matrix constants with damage.

Assuming cracks in the piezoelectric plate have the same direction and considering that the matrix cracks in the piezoelectric plate are parallel to the coordinate plane $1-3$, all damage variables except ω_2 are zero, the effect of the damage on the stiffness of plate in this coordinate plane $1-3$ can be neglected and $\sigma_3 = 0$ is forced in the status of plane stress, the relations of Eq.(1) can be written as

$$\begin{aligned} \sigma_p &= [C_{pq}^0 + C_{pq}]\varepsilon_q - [e_{pm}^0 + e_{pm}]E_m \overset{def}{=} [C_{pq}^* + \alpha_{pq}\omega_2]\varepsilon_q - [e_{pm}^* + \beta_{pm}\omega_2]E_m \\ D_m &= [e_{qm}^0 + e_{qm}]^T\varepsilon_q + [\kappa_m^0 + \kappa_m]E_m \overset{def}{=} [e_{qm}^* + \beta_{qm}\omega_2]^T\varepsilon_q + [\kappa_m^* + \gamma_m\omega_2]E_m \quad (p,q = 1,2,6 \;\; m = 1,2,3). \end{aligned} \quad (2)$$

where $C_{pq}^0, e_{pm}^0, \kappa_m^0$ are the same as before. The superscript $k = 1$ is omitted in $C_{pq}^k, e_{pm}^k, \kappa_m^k$.

For the piezoelectric plate, only thickness direction electric field E_z is dominant. If the voltage applied to the piezoelectric plate with piezoelectric effect in the thickness only, then $E_z = V/h$, where V is the applied voltage across the thickness of piezoelectric plate.

Introduce the following dimensionless parameters:

$$\begin{aligned} &\xi = \frac{x}{a},\ \eta = \frac{y}{b},\ U = \frac{u}{a},\ V = \frac{v}{b},\ W = \frac{w}{h},\ \bar{z} = \frac{z}{h}, \lambda_1 = \frac{h}{a}, \lambda_2 = \frac{h}{b}, \overline{C_{12}^d} = \frac{C_{12}^d}{C_{11}^*}, \overline{C_{22}^d} = \frac{C_{22}^d}{C_{11}^*}, \bar{\sigma}_{eq} = \frac{\sigma_{eq}}{C_{11}^*}, \bar{\sigma}_d = \frac{\sigma_d}{C_{11}^*}, \\ &\overline{e_{31}^d} = \frac{e_{31}^d}{e_{31}^*}, \overline{e_{32}^d} = \frac{e_{32}^d}{e_{31}^*}, \bar{\beta}_{32} = \frac{\beta_{32}}{e_{31}^*}, \tau = \frac{t}{h}\sqrt{\frac{C_{11}^*}{\rho}}, Q = \frac{q}{C_{11}^*}, \bar{V} = \frac{e_{31}^* V}{C_{11}^* h}, \bar{A}_{ij} = \frac{A_{ij}}{C_{11}^* h}, \bar{D}_{ij} = \frac{D_{ij}}{C_{11}^* h^3}, \bar{\alpha}_{ij} = \frac{\alpha_{ij}}{C_{11}^*}\ (i,j = 1,2,6) \end{aligned} \quad (3)$$

where $A_{ij}, D_{ij} (i,j = 1,2,6)$ are defined as follow:

$$A_{ij} = \int_{-h/2}^{h/2} (C_{ij}^* + \alpha_{ij}\omega_2)dz\,,\quad D_{ij} = \int_{-h/2}^{h/2} z^2(C_{ij}^* + \alpha_{ij}\omega_2)dz \quad (i,j = 1,2,6). \quad (4)$$

According to the classical nonlinear plate theory, the dimensionless nonlinear governing equations of motion for the piezoelectric plate with damage can be expressed in terms of U, V and W as follows:

$$\begin{aligned} &\bar{A}_{11,\xi}(U_{,\xi} + \tfrac{1}{2}\lambda_1^2 W_{,\xi}^2) + \bar{A}_{11}(U_{,\xi\xi} + \lambda_1^2 W_{,\xi}W_{,\xi\xi}) + \bar{A}_{12,\xi}(V_{,\eta} + \tfrac{1}{2}\lambda_2^2 W_{,\eta}^2) + \bar{A}_{12}(V_{,\xi\eta} + \lambda_2^2 W_{,\eta}W_{,\xi\eta}) \\ &+ \tfrac{\lambda_2}{\lambda_1}\bar{A}_{66,\eta}(\tfrac{\lambda_2}{\lambda_1}U_{,\eta} + \tfrac{\lambda_1}{\lambda_2}V_{,\xi} + \lambda_1\lambda_2 W_{,\xi}W_{,\eta}) + \bar{A}_{66}(\tfrac{\lambda_2^2}{\lambda_1^2}U_{,\eta\eta} + V_{,\xi\eta} + \lambda_2^2 W_{,\eta}W_{,\xi\eta} + \lambda_2^2 W_{,\xi}W_{,\eta\eta}) - \overline{e_{31}^d}_{,\xi}\bar{V} = 0 \\ &\bar{A}_{12,\eta}(U_{,\xi} + \tfrac{1}{2}\lambda_1^2 W_{,\xi}^2) + \bar{A}_{12}(U_{,\xi\eta} + \lambda_1^2 W_{,\xi}W_{,\xi\eta}) + \bar{A}_{22,\eta}(V_{,\eta} + \tfrac{1}{2}\lambda_2^2 W_{,\eta}^2) + \bar{A}_{22}(V_{,\eta\eta} + \lambda_2^2 W_{,\eta}W_{,\eta\eta}) \\ &+ \tfrac{\lambda_1}{\lambda_2}\bar{A}_{66,\xi}(\tfrac{\lambda_2}{\lambda_1}U_{,\eta} + \tfrac{\lambda_1}{\lambda_2}V_{,\xi} + \lambda_1\lambda_2 W_{,\xi}W_{,\eta}) + \bar{A}_{66}(U_{,\xi\eta} + \tfrac{\lambda_1^2}{\lambda_2^2}V_{,\xi\xi} + \lambda_1^2 W_{,\eta}W_{,\xi\xi} + \lambda_1^2 W_{,\xi}W_{,\xi\eta}) - \overline{e_{32}^d}_{,\xi}\bar{V} = 0 \end{aligned}$$

$$-\lambda_1^4(\bar{D}_{11,\xi\xi}W_{,\xi\xi}+2\bar{D}_{11,\xi}W_{,\xi\xi\xi}+\bar{D}_{11}W_{,\xi\xi\xi\xi})-\lambda_1^2\lambda_2^2(\bar{D}_{12,\xi\xi}W_{\eta\eta}+2\bar{D}_{12,\xi}W_{,\xi\eta\eta}+\bar{D}_{12}W_{,\xi\xi\eta\eta})-4\lambda_1^2\lambda_2^2$$
$$(\bar{D}_{66,\xi\eta}W_{,\xi\eta}+\bar{D}_{66,\xi}W_{,\xi\eta\eta}+\bar{D}_{66,\eta}W_{,\xi\xi\eta}+\bar{D}_{66}W_{,\xi\xi\eta\eta})-\lambda_1^2\lambda_2^2(\bar{D}_{12,\eta\eta}W_{,\xi\xi}+2\bar{D}_{12,\eta}W_{\xi\xi\eta}+\bar{D}_{12}W_{,\xi\xi\eta\eta})-$$
$$\lambda_2^4(\bar{D}_{22,\eta\eta}W_{,\eta\eta}+2\bar{D}_{22,\eta}W_{,\eta\eta\eta}+\bar{D}_{22}W_{,\eta\eta\eta\eta})+\lambda_1^2[\bar{A}_{11}(U_{,\xi}+\tfrac{1}{2}\lambda_1^2W_{,\xi}^2)+\bar{A}_{12}(V_{,\eta}+\tfrac{1}{2}\lambda_2^2W_{,\eta}^2)-\overline{e_{31}^d}\bar{V}]W_{,\xi\xi}$$
$$+2\lambda_1\lambda_2\bar{A}_{66}(\tfrac{\lambda_2}{\lambda_1}U_{,\eta}+\tfrac{\lambda_1}{\lambda_2}V_{,\xi}+\lambda_1\lambda_2W_{,\xi}W_{,\eta})W_{,\xi\eta}+\lambda_2^2[\bar{A}_{12}(U_{,\xi}+\tfrac{1}{2}\lambda_1^2W_{,\xi}^2)+\bar{A}_{22}(V_{,\eta}+\tfrac{1}{2}\lambda_2^2W_{,\eta}^2)-$$
$$\overline{e_{32}^d}\bar{V}]W_{,\eta\eta}+Q(\xi,\eta,\tau)=W_{,\tau\tau} \tag{5}$$

Suppose the boundary of the piezoelectric plate is simply supported, the dimensionless boundary conditions can be expressed as

$$\xi=0,1:\ V=0,\ \bar{A}_{11}(U_{,\xi}+\tfrac{1}{2}\lambda_1^2W_{,\xi}^2)-\overline{e_{31}^d}\bar{V}=0,\quad W=0,\ W_{,\xi\xi}=0$$
$$\eta=0,1:\ U=0,\ \bar{A}_{22}(V_{,\eta}+\tfrac{1}{2}\lambda_2^2W_{,\eta}^2)-\overline{e_{32}^d}\bar{V}=0,\quad W=0,\ W_{,\eta\eta}=0 \tag{6}$$

In the present research, Kachanvo damage evolution law[7] is adopted for an arbitrary point in the piezoelectric plate with damage and the dimensionless form can be written as

$$\frac{\partial\omega_2}{\partial\tau}=\begin{cases}\bar{B}(\dfrac{\bar{\sigma}_{eq}}{1-\omega_2})^n & \bar{\sigma}_{eq}\ge\bar{\sigma}_d\\ 0 & \bar{\sigma}_{eq}<\bar{\sigma}_d\end{cases} \tag{7}$$

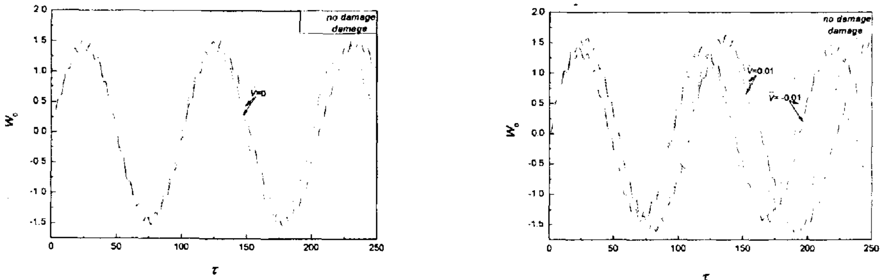
Taking the mid-surface normal stress of the piezoelectric plate as the equivalent stress σ_{eq}^k that is vertical to the fibrous direction, it can be presented as

$$\bar{\sigma}_{eq}=\overline{C_{12}^d}(U_{,\xi}+\tfrac{1}{2}\lambda_1^2W_{,\xi}^2-\lambda_1^2\bar{z}W_{,\xi\xi})+\overline{C_{22}^d}(V_{,\eta}+\tfrac{1}{2}\lambda_2^2W_{,\eta}^2-\lambda_2^2\bar{z}W_{,\eta\eta})-\overline{e_{32}^d}\bar{V}. \tag{8}$$

Solution Methodology and Numerical Results

Suppose the dimensionless transverse load is taken as $Q=f(\tau)\sin\pi\xi\sin\pi\eta$, $f(\tau)=f_0\sin\theta\tau$, where f_0 and θ represent the dimensionless amplitude and frequency of the load, respectively. To seek the approximate solutions of Eq.(5) which satisfied the boundary conditions (6), the unknown functions U, V and W are separated both for space and for time. The finite difference method and the Newmark scheme are used for space and time, respectively.

In all examples, the geometric parameters are given as $\lambda_1=0.1$, $\lambda_2=0.1$. The material parameters related to damage in all examples are taken as $\bar{B}=0.8, n=1.2$, $\bar{\sigma}_d=10^{-3}$, $\overline{\alpha_{11}}=-0.15, \overline{\alpha_{12}}=-0.15$, $\overline{\alpha_{22}}=-0.15$, $\overline{\alpha_{66}}=-0.15$, $\overline{\beta_{31}}=-0.15, \overline{\beta_{32}}=-0.15$.



(a) Without electric load (b) With electric load

Fig.1 Effect of damage on nonlinear dynamic response of piezoelectric plate